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Citation for published version:

Del Debbio, L, Faber, M, Greensite, J & Olejnik, S 2002, Center Dominance, Center Vortices, and Confinement. in *New Developments in Quantum Field Theory*. vol. 366, NATO Science Series: B:, vol. 366, Springer US, pp. 47-64. https://doi.org/10.1007/0-306-47075-6_4

Digital Object Identifier (DOI):

[10.1007/0-306-47075-6_4](https://doi.org/10.1007/0-306-47075-6_4)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Early version, also known as pre-print

Published In:

New Developments in Quantum Field Theory

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CENTER DOMINANCE, CENTER VORTICES, AND CONFINEMENT

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Invited talk, presented by J. Greensite, at the Nato Workshop “New Developments in Quantum Field Theory,” June 14-20, 1997, Zakopane, Poland

INTRODUCTION

The results that I would like to discuss here are a collection of numerical data which strongly favor an old and, in recent years, somewhat neglected theory of quark confinement: the Z_N Vortex Condensation Theory. Some of this data (Figs. 1-8) was reported by our group late last year,¹ the rest is new.

The confinement region of an $SU(N)$ gauge theory really consists of at least two parts. The first is an intermediate distance region, extending from the onset of the linear potential up to some color-screening distance, which we call the **Casimir-Scaling regime**.^{2, 3} Many numerical experiments have shown that in this intermediate region flux tubes form, and a linear potential is established, between heavy quarks in any non-trivial representation of the gauge group. The string-tension is representation-dependent, and appears to be roughly proportional to the quadratic Casimir of the representation.⁴ Thus, for an $SU(2)$ gauge theory,

$$\sigma_j \approx \frac{4}{3}j(j+1)\sigma_{1/2} \quad (1)$$

where σ_j is the string tension for a heavy quark-antiquark pair in representation j . Eventually, however, the color charge of higher-representation quarks must be screened by gluons, and the asymptotic string tension can then only depend on the transformation properties of the quarks under the center of the gauge group, i.e. on the “n-ality” of

the representation. This **Asymptotic regime** extends from the color-screening length to infinity, and in the case of an $SU(2)$ gauge group the string tensions must satisfy

$$\sigma_j = \begin{cases} \sigma_{1/2} & j = \text{half-integer} \\ 0 & j = \text{integer} \end{cases} \quad (2)$$

In particular, the string between quarks in an adjoint representation must break, at some distance which presumably depends on the mass of "gluelumps" (i.e. the energy of a gluon bound to a massive adjoint quark). Also, since string-breaking is a $1/N^2$ suppressed process, the number of colors is relevant. The breaking of the adjoint string is difficult to observe in numerical experiments, although on general theoretical grounds one may be confident that the breaking *must* occur for sufficiently large adjoint quark separation.

The most popular theory of quark confinement is the abelian projection theory proposed by 't Hooft, which I will briefly describe in the next section. In past years our group has been highly critical of this theory (as well as the Z_N vortex theory), on the grounds that it fails to explain the existence of a linear potential between higher representation quarks in the Casimir scaling regime.^{2, 3} This failure is very significant, because it is in the Casimir regime that the confining force replaces Coulombic behavior, and in fact it is only in this regime that the QCD string has been well studied numerically. If we don't understand Casimir scaling, then we don't really understand how flux tubes form.

A possible response to this criticism is simply to admit that the formation of flux tubes, at intermediate distances, remains to be understood, but that the abelian projection theory is nonetheless valid at very large distance scales, i.e. in the asymptotic regime. I will argue that there may be some truth to this response, but that the confining configurations relevant to the asymptotic regime seem to be Z_N vortices, rather than abelian monopoles.

ABELIAN DOMINANCE

One of the earliest ideas about confinement, known as "dual-superconductivity," was put forward independently by 't Hooft and Mandelstam in the mid-1970's. The idea is that the QCD vacuum resembles a superconductor with the roles of the \vec{E} and \vec{B} fields interchanged. Electric (rather than magnetic) fields are squeezed into vortices; electric (rather than magnetic) charges are confined. Magnetic monopoles are condensed; they play the role of the electrically charged Cooper pairs. The problem is to actually identify the magnetic monopoles of an unbroken non-abelian gauge theory, and to understand which non-abelian degrees of freedom play the role of electromagnetism.

A concrete suggestion along these lines was made by 't Hooft in 1981.⁵ The proposal was to gauge fix part of the $SU(N)$ symmetry by diagonalizing some operator transforming in the adjoint representation of the gauge group. This leaves a remnant $U(1)^{N-1}$ gauge symmetry, with gauge transformations g of the form

$$g = \text{diag}[e^{i\alpha_1}, e^{i\alpha_2}, \dots, e^{i\alpha_N}] \quad \sum \alpha_n = 0 \quad (3)$$

The diagonal components of the vector potential, \mathcal{A}_μ^{aa} , transform under the residual symmetry like abelian gauge fields, i.e.

$$\mathcal{A}_\mu'^{aa} = \mathcal{A}_\mu^{aa} + \partial_\mu \alpha^a \quad (4)$$

while the off-diagonal components transform like double (abelian) charged matter fields

$$\mathcal{A}_\mu'^{ab} = e^{i(\alpha^a - \alpha^b)} \mathcal{A}_\mu^{ab} \quad (5)$$

This gauge-fixed theory can therefore be regarded as an abelian gauge theory of “photons,” charged matter fields, and magnetic monopoles. Monopole condensation confines abelian charged objects, and the abelian electric field forms a flux tube.

On the lattice, one can decompose the link variables U

$$U_\mu(x) = W_\mu(x) A_\mu(x) \quad (6)$$

into “abelian” link variables A , transforming under the residual symmetry as abelian gauge fields, and “matter” fields W

$$\begin{aligned} A'_\mu(x) &= g(x) A_\mu(x) g^{-1}(x + \hat{\mu}) \\ W'_\mu(x) &= g(x) W_\mu(x) g^{-1}(x) \end{aligned} \quad (7)$$

For $SU(2)$ lattice gauge theory, A is simply the diagonal part of U , rescaled to restore unitarity, i.e.

$$U = a_0 I + i \vec{a} \cdot \vec{\sigma} \quad , \quad A = \frac{a_0 + i a_3 \sigma^3}{\sqrt{a_0^2 + a_3^2}} \quad (8)$$

Monte Carlo studies of the abelian projection theory began with the work of Kronfeld et al.,⁶ who introduced a specific abelian projection gauge, the “maximal abelian gauge,”⁷ which has been used in most further studies. The maximal abelian gauge is defined as the gauge which maximizes the quantity

$$\sum_x \sum_\mu \text{Tr}[U_\mu(x) \sigma^3 U_\mu^\dagger(x) \sigma^3] \quad (9)$$

This requires diagonalizing, at every site, the adjoint representation operator

$$X(x) = \sum_\mu [U_\mu(x) \sigma^3 U_\mu^\dagger(x) + U_\mu^\dagger(x - \hat{\mu}) \sigma^3 U_\mu(x - \hat{\mu})] \quad (10)$$

This gauge choice makes the link variables as diagonal as possible, placing most of the quantum fluctuations in the abelian link variables. If the abelian projection idea is going to work at all, it ought to work best in this gauge. Other proposals (Polyakov-line gauge, Field-Strength gauge) have not, in fact, been very successful.

An important development was the finding, by Suzuki and collaborators, that if we fix to maximal abelian gauge and replace the full link variables U with the abelian link variables A (this is often termed “abelian projection”), and then calculate such quantities as Creutz ratios, Polyakov lines, etc., with the abelian links, the results very closely approximate those obtained with the full link variables.⁸ The fact that the abelian link variables seem to carry most of the information about the infrared physics is known as “**abelian dominance**,” and it has stimulated a great deal of further work on the abelian projection theory.

CENTER DOMINANCE

Of course the abelian projection theory is not the *only* proposal for explaining the confining force; there have been many other suggestions over the years. One idea that was briefly popular in the late 1970's was the Vortex Condensation theory, put forward, in various forms, by 't Hooft,⁹ Mack,¹⁰ and by Nielsen and Olesen¹¹ (the “Copenhagen Vacuum”). The idea is that the QCD vacuum is filled with closed magnetic vortices, which have the topology of tubes (in 3 Euclidean dimensions) or surfaces (in 4 dimensions) of finite thickness, and which carry magnetic flux in the center of the gauge group (hence “center vortices”). The effect of creating a center vortex linked to a given Wilson loop, in an $SU(N)$ gauge theory, is to multiply the Wilson loop by an element of the gauge group center, i.e.

$$W(C) \rightarrow e^{i2\pi n/N} W(C) \quad n = 1, \dots, N-1 \quad (11)$$

Quantum fluctuations in the number of vortices linked to a Wilson loop can be shown to lead to an area law falloff, assuming that center vortex configurations are condensed in the vacuum.*

With one notable exception,¹³ almost nothing has been done with this idea since the early 1980's, which was at the dawn of Monte Carlo lattice gauge simulations. It is therefore interesting to go back and study the vortex theory, using a numerical approach inspired by studies of the abelian projection theory.

In an $SU(2)$ lattice gauge theory, we begin by fixing to maximal abelian gauge and then go one step further, using the remnant $U(1)$ symmetry to bring the abelian link variables

$$A = \begin{bmatrix} e^{i\theta} & \\ & e^{-i\theta} \end{bmatrix} \quad (12)$$

as close as possible to the $SU(2)$ center elements $\pm I$ by maximizing $\langle \cos^2 \theta \rangle$, leaving a remnant Z_2 symmetry. This is the **(indirect) Maximal Center Gauge** (the center is maximized in A , rather than directly in U). We then define at each link

$$Z \equiv \text{sign}(\cos \theta) = \pm 1 \quad (13)$$

which is easily seen to transform like a Z_2 gauge field under the remnant Z_2 symmetry. “Center Projection” is the replacement $U \rightarrow Z$ of the full link variables by the center variables; we can then calculate Wilson loops, Creutz ratios, etc. with the center-projected Z -link variables.

Figure 1 is a plot of Creutz ratios vs. β , extracted from the center-projected configurations. The straight line is the asymptotic freedom prediction

$$\sigma a^2 = \frac{\sigma}{\Lambda^2} \left(\frac{6\pi^2}{11} \beta \right)^{102/121} \exp \left[-\frac{6\pi^2}{11} \beta \right] \quad (14)$$

with the value $\sqrt{\sigma}/\Lambda = 67$. What is remarkable about this plot, apart from the scaling, is that the Creutz ratios $\chi(R, R)$ at each β are almost independent of R . This

*Some related ideas have also been put forward by Chernodub et al.¹²

means that the center projection sweeps away the Coulombic contribution, and the linear potential appears already at $R = 2$. This is seen quite clearly in Fig. 2, which compares the center-projected Creutz ratios (solid line), at $\beta = 2.4$, with the Creutz ratios of the full theory (dashed line). It is also interesting to compute the Creutz ratios derived from abelian link variables with the Z variable factored out, i.e. A/Z (dotted line). We note that, in this case, the string tension simply disappears.

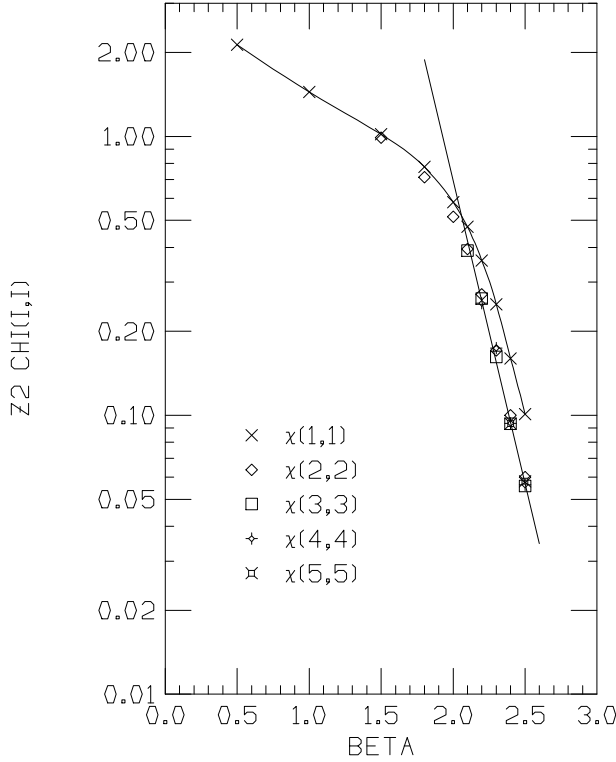


Figure 1. Creutz ratios from center-projected lattice configurations, in the (indirect) maximal center gauge.

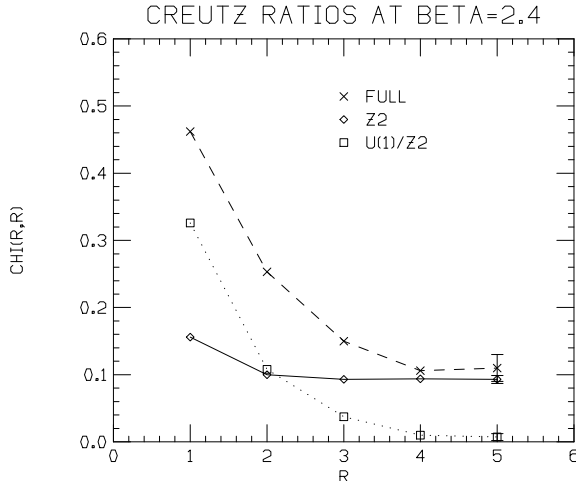


Figure 2. Creutz ratios $\chi(R, R)$ vs. R at $\beta = 2.4$ for full, center-projected, and $U(1)/Z_2$ -projected lattice configurations.

It seems evident from this data that, just as the abelian A links are the crucial part of the full U link variables in maximal abelian gauge, so the Z center variables are the crucial part of the A links in maximal center gauge, carrying most of the information about the string tension. This is what we mean by “Center Dominance.”

Should one then interpret center dominance to mean that the confining force is due to Z_2 center vortices, rather than $U(1)$ monopoles? That conclusion would be premature, in our view. In fact, our original interpretation of this data was that the success of center dominance suggests that *neither* abelian dominance *nor* center dominance has anything very convincing to say about quark confinement (and this fits very nicely with our further critique of abelian projection based on Casimir scaling).² Underlying that interpretation, however, was the belief that the “thin” Z_2 vortices of the center-projected configurations are probably irrelevant to the confining properties of the full, unprojected configurations. This belief is testable, however, and the result of the test is surprising.

VORTEX-LIMITED WILSON LOOPS

The only excitations of Z_2 lattice gauge theory with non-zero action are “thin” Z_2 vortices, which have the topology of a surface (one lattice spacing thick) in $D=4$ dimensions. We will call the Z_2 vortices, of the center projected Z -link configurations, “Projection-vortices” or just **P-vortices**. These are to be distinguished from the hypothetical “thick” center vortices, which might exist in the full, unprojected U configurations. A plaquette is pierced by a P-vortex if, upon going to maximal center gauge and center-projecting, the projected plaquette has the value -1 . Likewise, a given lattice surface is pierced by n P-vortices if n plaquettes of the surface are pierced by P-vortices.

In a Monte Carlo simulation, the number of P-vortices piercing the minimal area of a given loop C will, of course, fluctuate. Let us define $W_n(C)$ to be the Wilson loop evaluated on a sub-ensemble of configurations, selected such that precisely n P-vortices, in the corresponding center-projected configurations, pierce the minimal area of the loop. It should be emphasized here that the center projection is used only to select the data set. The Wilson loops themselves are evaluated using the full, *unprojected* link variables. In practice, to compute $W_n(C)$, the procedure is to generate thermalized lattice configurations by the usual Monte Carlo algorithm, and fix to maximal center gauge by over-relaxation. For each independent configuration one then examines each rectangular loop on the lattice of a given size; those with n P-vortices piercing the loop are evaluated, the others are skipped. Creutz ratios $\chi_n(I, J)$ can then be extracted from the vortex-limited Wilson loops $W_n(C)$. In particular, if the presence or absence of P-vortices in the projected configuration is unrelated to the confining properties of the corresponding unprojected configuration, then we would expect

$$\chi_0(I, J) \approx \chi(I, J) \tag{15}$$

at least for large loops.

The result of this test is shown in Fig. 3. Quite contrary to our expectations, the confining force vanishes if P-vortices are excluded. This does not necessarily mean that

the confining configurations of $SU(2)$ lattice gauge theory are thick center vortices. It does imply, however, that the presence or absence of P-vortices in the projected gauge field is strongly correlated with the presence or absence of confining configurations (whatever they may be) in the unprojected gauge field.

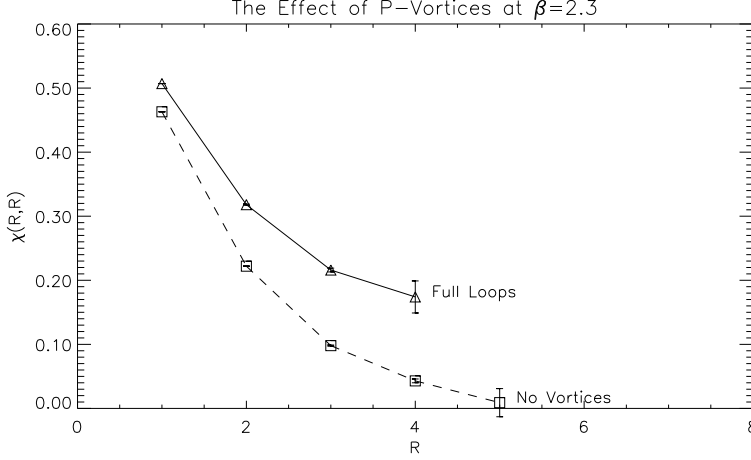


Figure 3. Creutz ratios $\chi_0(R, R)$ extracted from loops with no P-vortices, as compared to the usual Creutz ratios $\chi(R, R)$, at $\beta = 2.3$.

The next question is whether we can rule out the possibility that the confining configurations are, in fact, thick Z_2 center vortices. Suppose, for a moment, that to each P-vortex in the projected Z -link gauge field there corresponds a thick center vortex in the associated, unprojected, U -link gauge field. If that is the case, then in the limit of large loop area we expect

$$\frac{W_n(C)}{W_0(C)} \longrightarrow (-1)^n \quad (16)$$

The argument for this equation is as follows: Vortices are created by discontinuous gauge transformations. Suppose loop C , parametrized by $x^\mu(\tau)$, $\tau \in [0, 1]$, encircles n vortices. At the point of discontinuity

$$g(x(0)) = (-1)^n g(x(1)) \quad (17)$$

The corresponding vector potential, in the neighborhood of loop C can be decomposed as

$$A_\mu^{(n)}(x) = g^{-1} \delta A_\mu^{(n)}(x) g + i g^{-1} \partial_\mu g \quad (18)$$

so that

$$\begin{aligned} W_n(C) &= \langle \text{Tr} \exp[i \oint dx^\mu A_\mu^{(n)}] \rangle \\ &= (-1)^n \langle \text{Tr} \exp[i \oint dx^\mu \delta A_\mu^{(n)}] \rangle \end{aligned} \quad (19)$$

In the region of the loop C , the vortex background looks locally like a gauge transformation. If all other fluctuations $\delta A_\mu^{(n)}$ are basically short-range, then they should

be oblivious, in the neighborhood of the loop C , to the presence or absence of vortices in the middle of the loop. In that case, if we have correctly identified the vortex contribution, then

$$\langle \text{Tr exp}[i \oint dx^\mu \delta A_\mu^{(n)}] \rangle \approx \langle \text{Tr exp}[i \oint dx^\mu \delta A_\mu^{(0)}] \rangle \quad (20)$$

for sufficiently large loops, and eq. (16) follows immediately. All we have to do is test this.

Figures 4 and 5 show our data for W_1/W_0 and W_2/W_0 , respectively, at $\beta = 2.3$. Again, somewhat to our surprise, this data is entirely consistent with (16); it is consistent with the confining field configurations being center vortices, and in fact offers good evidence in favor of that possibility.

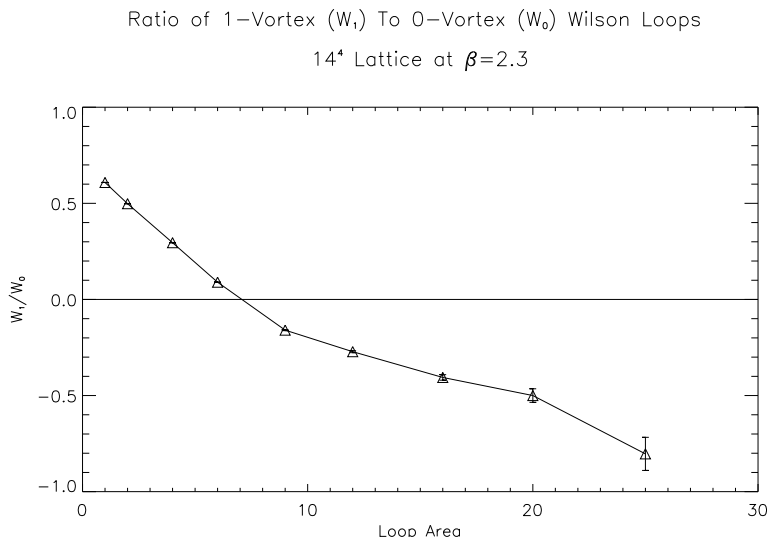


Figure 4. Ratio of the 1-Vortex to the 0-Vortex Wilson loops, $W_1(C)/W_0(C)$, vs. loop area at $\beta = 2.3$.

Of course, it could still be that we are looking at a rather small (and perhaps misleading) sample of the data, at least for the larger loops. Large loops will tend to be pierced by large numbers of P-vortices. As the area of a loop increases, the fraction of configurations in which no P-vortex (or exactly one, or exactly two P-vortices) pierces the loop will decrease, tending to zero in the limit. So let us instead consider $W_{evn}(C)$ and $W_{odd}(C)$, where $W_{evn}(C)$ denotes Wilson loops evaluated in configurations with an even (including zero) number of P-vortices piercing the loop, and $W_{odd}(C)$ denotes the corresponding quantity for odd numbers. Then

$$W(C) = P_{evn}(C)W_{evn}(C) + P_{odd}(C)W_{odd}(C) \quad (21)$$

where

$P_{evn}(C)$ = the fraction of configurations with an even (or zero)
number of P-vortices piercing loop C

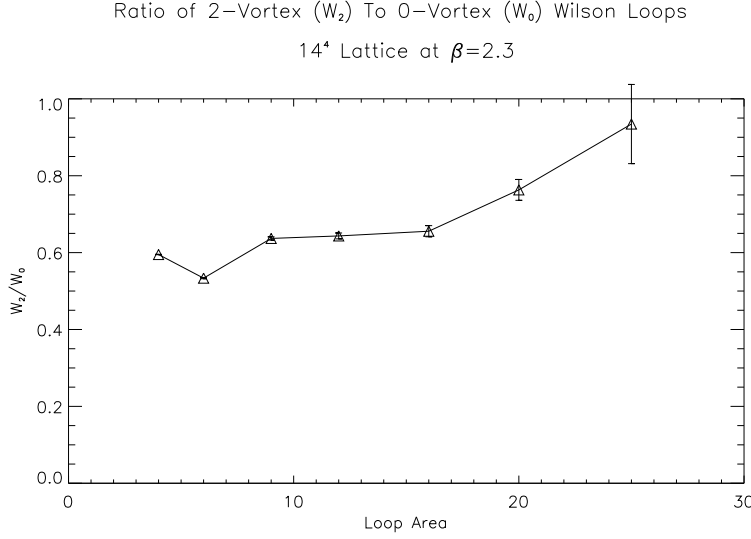


Figure 5. Ratio of the 2-Vortex to the 0-Vortex Wilson loops, $W_2(C)/W_0(C)$, vs. loop area at $\beta = 2.3$.

$P_{odd}(C)$ = the fraction of configurations with an odd
number of P-vortices piercing loop C

One expects that for large loops, $P_{evn} \approx P_{odd} \approx 0.5$. According to the vortex condensation mechanism, neither W_{evn} nor W_{odd} falls with an area law; the area-law falloff is due to a delicate cancellation between the two terms in eq. (21). As loops become large, one should find $W_{odd} \rightarrow -W_{evn}$. The data, shown below in Figures 6-8, support these expectations. This time we are using essentially all of the data, since about half contributes to $W_{evn}(C)$, and the rest to $W_{odd}(C)$.

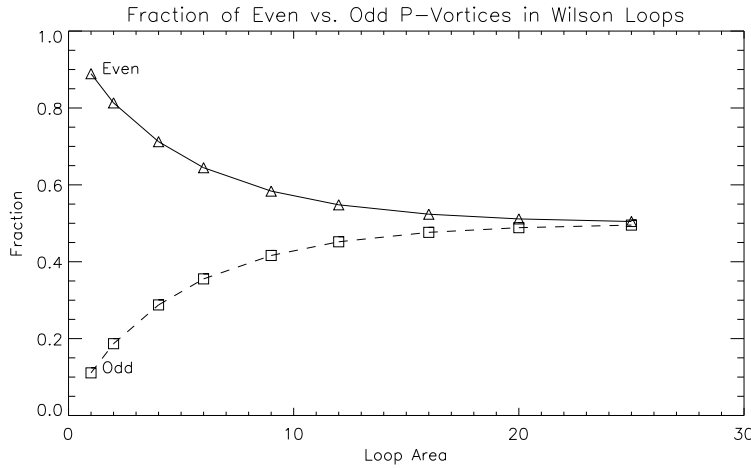


Figure 6. Fraction of link configurations containing even/odd numbers of P-vortices, at $\beta = 2.3$, piercing loops of various areas.

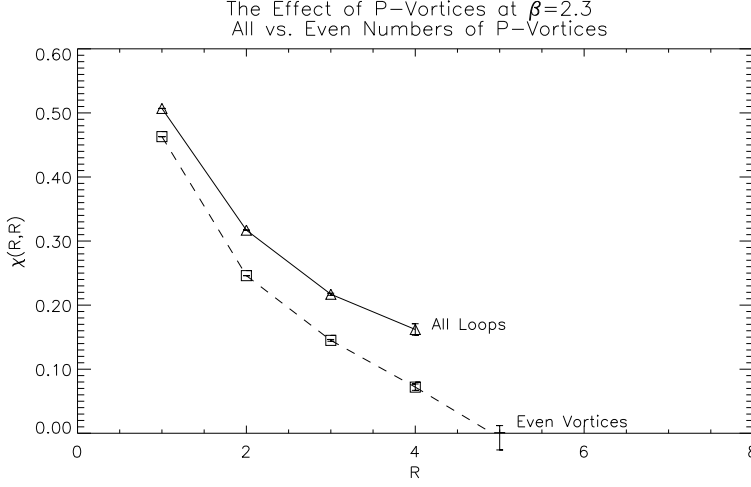


Figure 7. Creutz ratios $\chi_{evn}(R, R)$ extracted from Wilson loops $W_{evn}(C)$, taken from configurations with even numbers of P-vortices piercing the loop. The standard Creutz ratios $\chi(R, R)$ at this coupling ($\beta = 2.3$) are also shown.

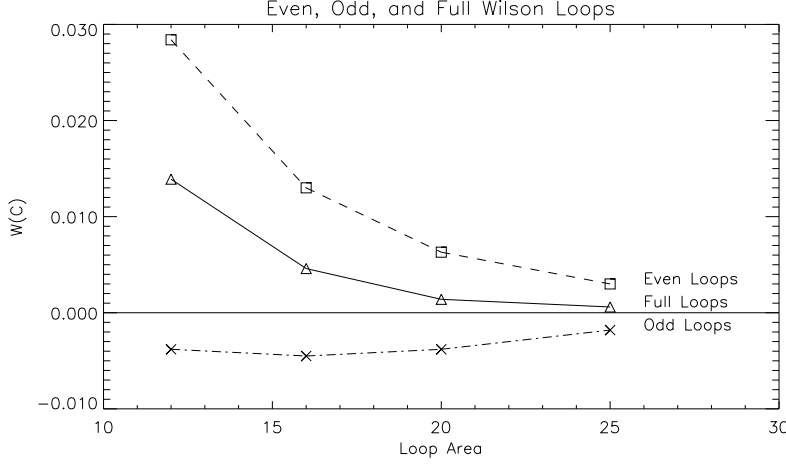


Figure 8. Wilson loops $W_{evn}(C)$, $W_{odd}(C)$ and $W(C)$ at larger loops areas, taken from configurations with even numbers of P-vortices, odd numbers of P-vortices, and any number of P-vortices, respectively, piercing the loop. Again $\beta = 2.3$.

DIRECT MAXIMAL CENTER GAUGE

Along with the successes, there is one significant failure of center dominance in the data shown in Fig. 1. Despite the nice scaling of the data, the value of $\sqrt{\sigma}/\Lambda = 67$ is a little high, and in fact the center projected Creutz ratios are all significantly higher than the asymptotic string tension extracted from unprojected configurations, using “state-of-the-art” noise reduction techniques.

On the other hand, it is not so clear that the “indirect” maximal center gauge is the *true* maximal center gauge. What we have done up until now is to first fix to maximal abelian gauge, and then bring the abelian part A of link U as close as possible to $\pm I$. However, since we are emphasizing the role of the gauge group center, rather than the $U(1)$ subgroup, it really makes more sense to choose a gauge in which the entire link variable U is brought as close as possible to the center elements $\pm I$. With this motivation, let us define the **(direct) Maximal Center Gauge** of an $SU(N)$ gauge theory as the gauge which maximizes the quantity

$$Q = \sum_x \sum_\mu \text{Tr}[U_\mu(x)] \text{Tr}[U_\mu^\dagger(x)] \quad (22)$$

For the $SU(2)$ gauge group, we define

$$Z = \text{sign}(\text{Tr}[U]) \quad (23)$$

as the center-projected link variables; these again transform like Z_2 gauge fields under the remnant Z_2 gauge symmetry.

Using the direct maximal center gauge, we find the following results: Qualitatively, things look about the same, and plots of W_n/W_0 , and W_{evn} vs. W_{odd} , look virtually identical to the previous data in the indirect maximal center gauge. Quantitatively, however, there is an improvement. We find that string tensions extracted from the center projection in the “direct” gauge are in much better agreement with the asymptotic string tension of the full theory, extracted by “state-of-the-art” methods. Figure 9 shows a plot of Creutz ratios vs. β . The straight line is the usual scaling curve, but this time with a value $\sqrt{\sigma}/\Lambda = 58$. Figures 10-12 plot the center-projected Creutz ratios $\chi(R, R)$ at $\beta = 2.3, 2.4, 2.5$ respectively. The triangles are our data. The solid line is the asymptotic string tension of the unprojected configurations at these values of β , quoted by Bali et al.¹⁴ The dashed lines are the error bars on the asymptotic string tension, which we have also taken from this reference.

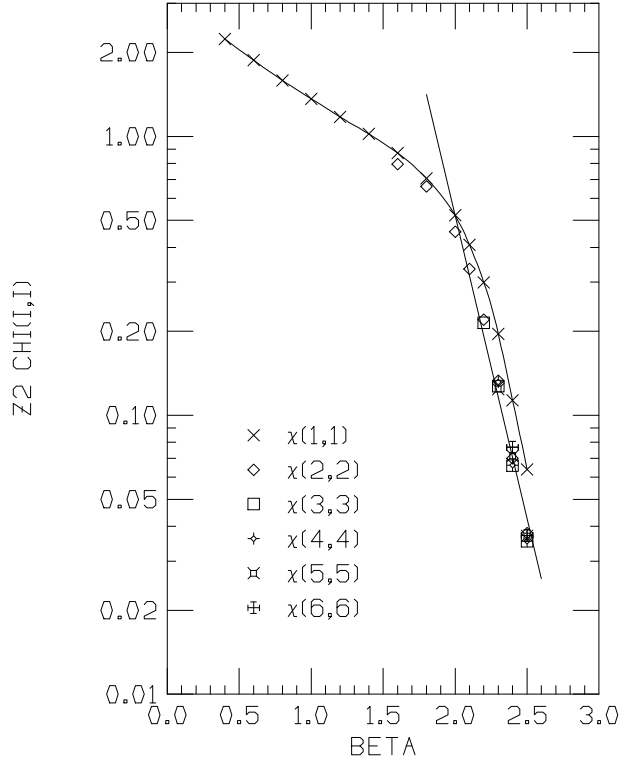


Figure 9. Creutz ratios from center-projected lattice configurations, in the direct maximal center gauge.

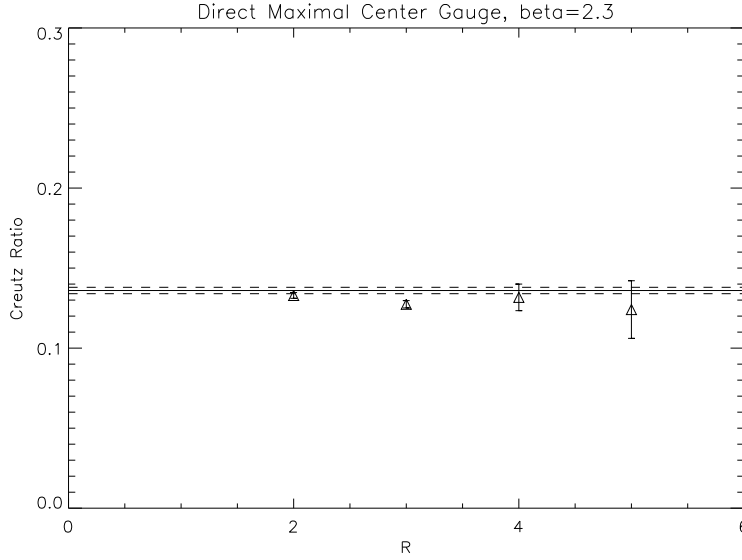


Figure 10. Center-projection Creutz ratios $\chi(R, R)$ vs. R at $\beta = 2.3$; direct maximal center gauge. Triangles are our data points. The solid line shows the value of the asymptotic string tension of the unprojected configurations, and the dashed lines the associated error bars, quoted in Bali et al.¹⁴

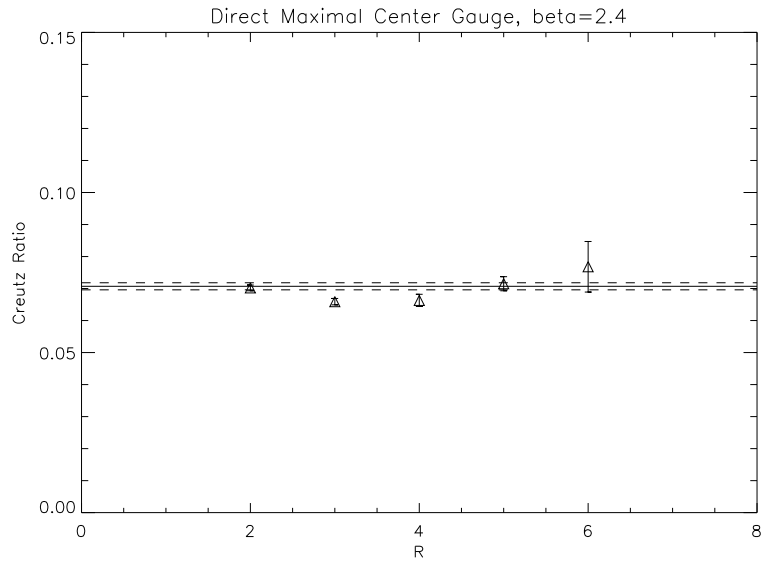


Figure 11. Same as Fig. 10, at $\beta = 2.4$.

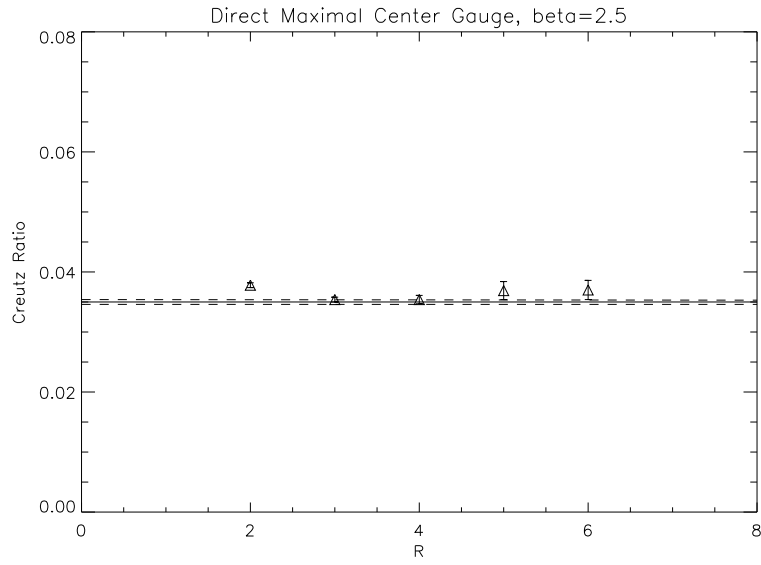
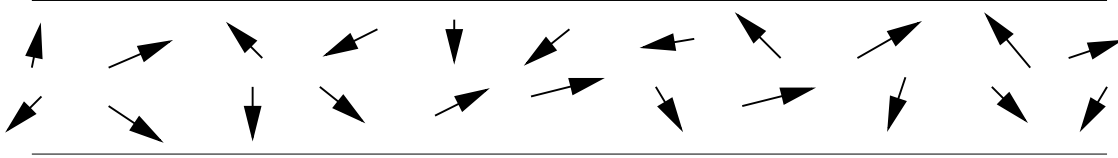


Figure 12. Same as Fig. 10, at $\beta = 2.5$.

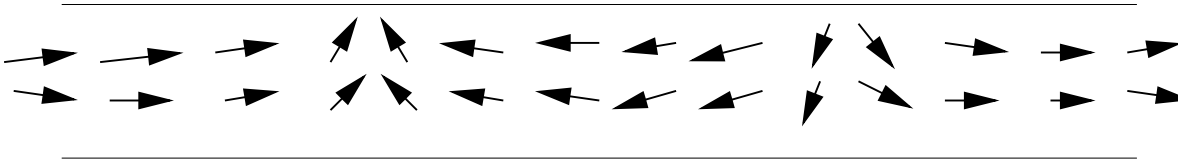
VORTICES VS. MONOPOLES

There is no denying that the data shown here, in support of the vortex condensation theory, is a little reminiscent of the data that has been put forward in support of the abelian projection theory. This raises a natural question: If the Yang-Mills vacuum is dominated, at long wavelengths, by Z_2 vortex configurations, then how do we explain the numerical successes of the abelian projection in maximal abelian gauge? In our opinion, the probable answer to this question is that a center vortex configuration, transformed to maximal abelian gauge and then abelian-projected, will appear as a chain of monopoles alternating with antimonopoles. These monopoles are essentially an artifact of the projection; they are condensed because the long vortices from which they emerge are condensed.

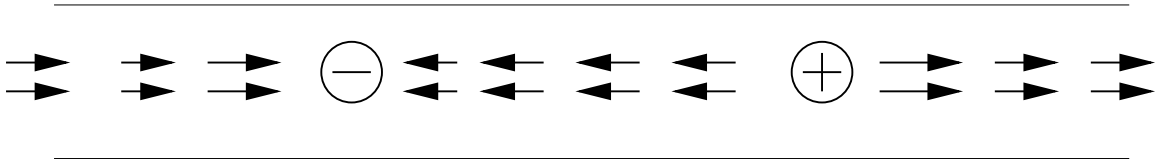
A little more graphically, the picture is as follows: Consider a center vortex at some constant time t . This time-slice of a thick vortex is then a tube of magnetic flux. Before gauge-fixing, the field-strength inside this tube points in arbitrary directions in color space



Fixing to maximal abelian gauge, the field strength tends to line up mainly (but not entirely) in the diagonal ($\pm\sigma^3$) color direction

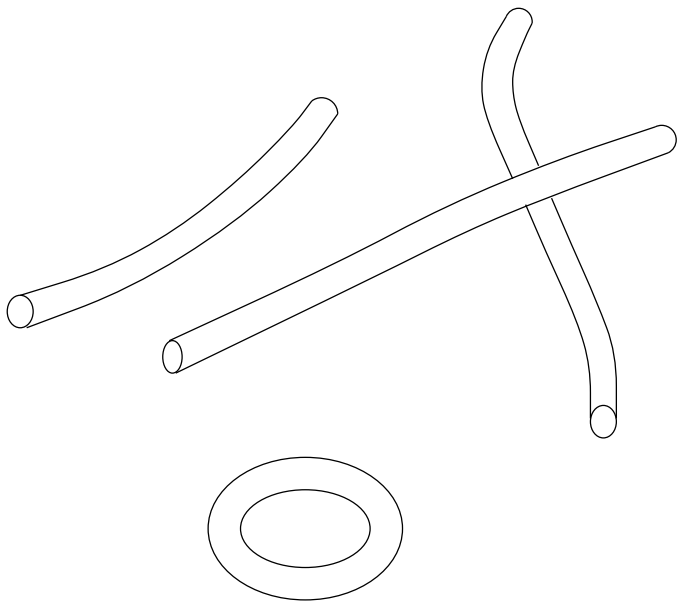


Upon abelian projection, the regions interpolating between $+\sigma^3$ and $-\sigma^3$ emerge as “monopoles.” Their location is gauge (and Gribov copy) dependent.

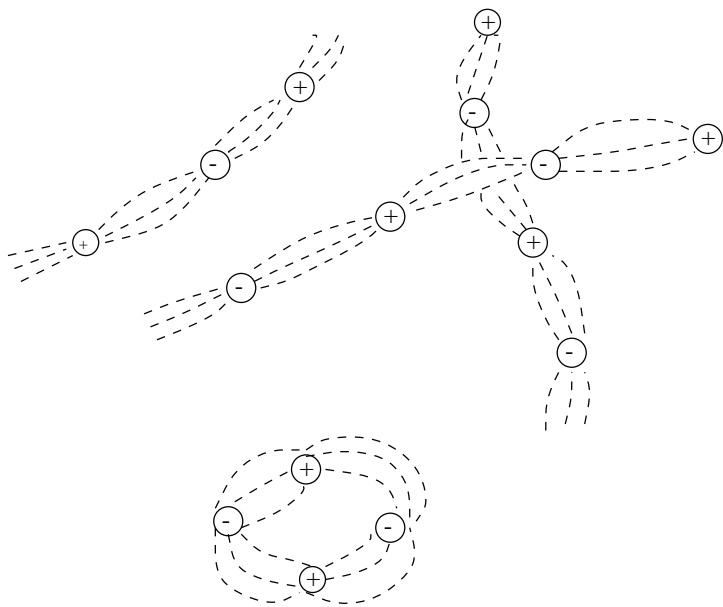


It is not difficult to construct examples of center vortices which behave in just this way, i.e. which are converted to monopole-antimonopole chains upon abelian projection in maximal abelian gauge.

If this picture is accurate, then the “spaghetti vacuum”

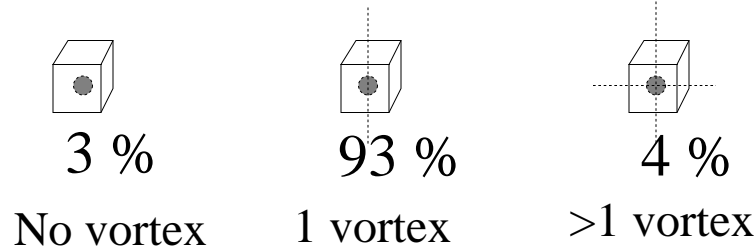


appears, under abelian projection, as a “monopole vacuum”



We have, in fact, obtained some preliminary evidence for this picture from Monte Carlo simulations. These simulations were carried out at $\beta = 2.4$, in the (indirect) maximal center gauge. We look at sites where the monopoles are “static,” i.e. the monopole current is $j_0 = \pm 1$, $\vec{j} = 0$. The monopole charge is enclosed in a cube bounded by spacelike plaquettes. We find that:

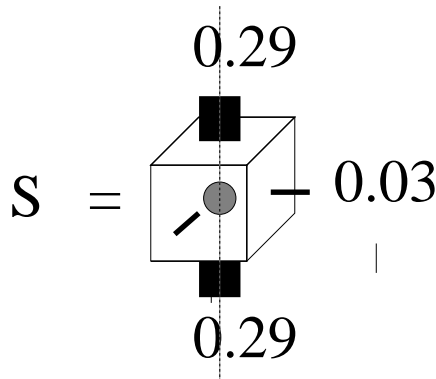
I: Almost all (93%) of monopole cubes are pierced by one, and only one, P-vortex.



II: The action of a monopole cube, pierced by a P-vortex, is highly asymmetric. Almost all the plaquette action

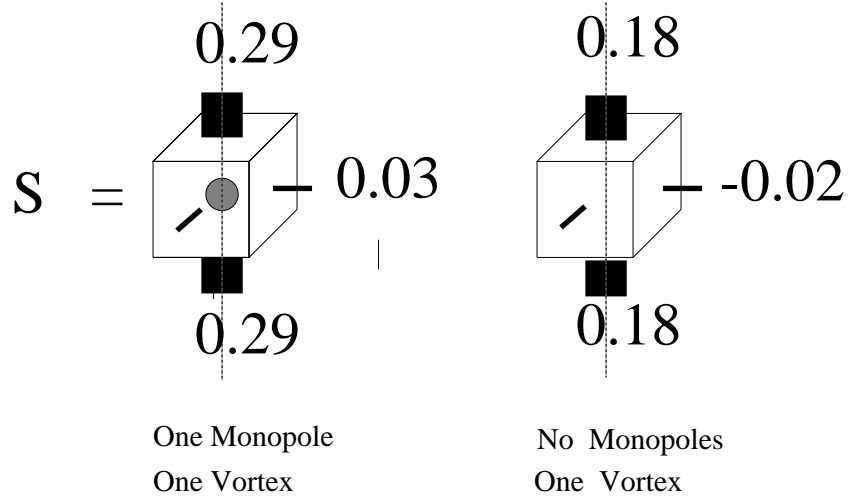
$$S = (1 - \frac{1}{2}\text{Tr}[UUU^\dagger U^\dagger]) - S_0 \quad (24)$$

above the lattice average S_0 , is oriented in the direction of the P-vortex. On each of the two plaquettes pierced by the P-vortex, at $\beta = 2.4$, the average action above S_0 is $S = 0.29$. On each of the four plaquettes which are not pierced by the vortex, $S = 0.03$ on average.[†]

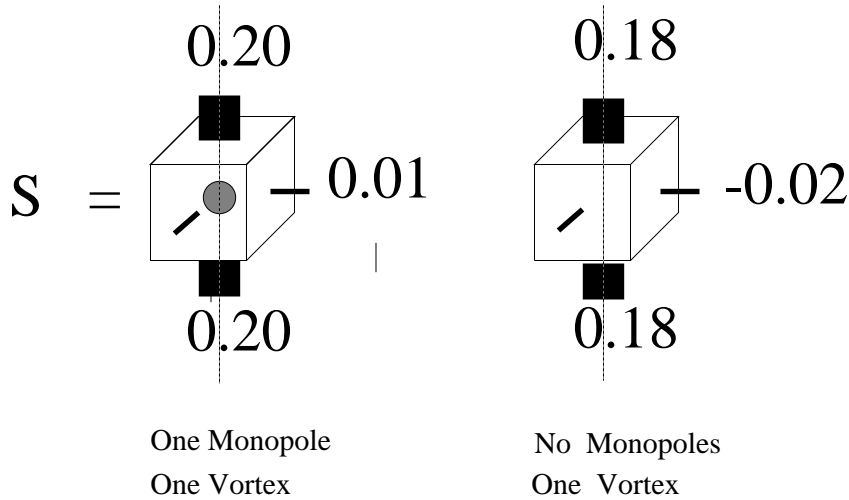


[†]Bakker et al.¹⁵ have also studied the excess action of monopole cubes (but not the correlation with P-vortices) in maximal abelian gauge.

III: The (unprojected) action distribution of a monopole cube, pierced by a P-vortex, is similar to the action distribution of any other cube pierced by a P-vortex...



...especially when we look at “isolated” monopoles (no neighboring monopole currents)



In summary, abelian monopoles tend to lie along P-vortices. Isolated monopoles are hardly distinguished, in their (unprojected) field strength distribution, from other regions along the P-vortices.

This is in accordance with our intuitive picture.

CONCLUSIONS

We have developed a technique for locating center vortices in thermalized lattice gauge configurations, and have found evidence that center vortices account for the asymptotic string tension between static, fundamental representation, color charges. A “spaghetti vacuum” picture appears to be correct at sufficiently large scales.

On the other hand, string formation at intermediate distances, in the Casimir scaling regime, remains to be understood. This is a very important issue, especially since the Casimir scaling regime extends to infinity as $N_{colors} \rightarrow \infty$.¹⁶ Casimir scaling suggests that center vortices, although they may be the crucial configurations asymptotically, are not the whole story. Since adjoint loops are oblivious to the gauge-group center, one may speculate that there are other types of configurations which contribute to the adjoint string tension. Or, possibly, the finite thickness and detailed inner structure of center vortices is a relevant issue, since adjoint loops which intersect the “core” of a center vortex *will* be affected by the vortex. Perhaps the gluon-chain model,¹⁷ which I proposed some time ago, might be helpful in understanding the dynamics of the Casimir-scaling region.

We are currently in the process of repeating all our calculations for $SU(3)$ lattice gauge theory, and have already found evidence of center dominance on small lattices at strong couplings. If we also find that (i) center dominance persists on larger lattices at weaker couplings; (ii) the absence of P-vortices results in vanishing string tension, and (iii)

$$\frac{W_n(C)}{W_0(C)} \longrightarrow e^{2n\pi i/3} \quad (25)$$

then the combined evidence in favor of some version of the Z_N vortex condensation theory will be quite compelling.

One final note: Shortly after the Zakopane meeting, Tomboulis and Kovács reported on some new Monte Carlo data they have obtained in support of the vortex condensation theory.¹⁸ Their results are quite consistent with the work I have presented here.

ACKNOWLEDGEMENTS

I would like to thank the organizers of the “New Developments” meeting for inviting me to Zakopane. I am also grateful for the hospitality of the high-energy theory group at Lawrence Berkeley National Laboratory, where some of this work was carried out. This research was supported in part by the U.S. Department of Energy, under Grant No. DE-FG03-92ER40711. Support has also been provided by Carlsbergfondet.

REFERENCES

1. L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, Phys. Rev. D55 (1997) 2298, hep-lat/9610005.
2. L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, Nucl. Phys. Proc. Suppl. 53 (1997) 141, hep-lat/9607053.

3. L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, Phys. Rev. D53 (1996) 5897, hep-lat/9510028.
4. J. Ambjørn, P. Olesen, and C. Peterson, Nucl. Phys. B240 [FS12] (1984) 198; 533;
C. Michael, Nucl. Phys. Proc. Suppl. 26 (1992) 417; Nucl. Phys. B259 (1985) 58;
N. Cambell, L. Jorysz, and C. Michael, Phys. Lett. B167 (1986) 91;
M. Faber and H. Markum, Nucl. Phys. Proc. Suppl. 4 (1988) 204;
M. Müller, W. Beirl, M. Faber, and H. Markum, Nucl. Phys. Proc. Suppl. 26 (1992) 423;
G. Poulis and H. Trottier, hep-lat/9504015.
5. G. 't Hooft, Nucl. Phys. B190 [FS3] (1981) 455.
6. A. Kronfeld, G. Schierholz, and U.-J. Wiese, Nucl. Phys. B293 (1987) 461.
7. A. Kronfeld, M. Laursen, G. Schierholz, and U.-J. Wiese, Phys. Lett. B198 (1987) 516.
8. T. Suzuki and Yotsuyanagi, Phys. Rev. D42 (1990) 4257;
S. Hioki et al., Phys. Lett. B272 (1991) 326.
9. G. 't Hooft, Nucl. Phys. B153 (1979) 141.
10. G. Mack, in *Recent Developments in Gauge Theories*, edited by G. 't Hooft et al. (Plenum, New York, 1980).
11. H. Nielsen and P. Olesen, Nucl. Phys. B160 (1979) 380.
12. M. Chernodub, M. Polikarpov, and M. Zubkov, Nucl. Phys. Proc. Suppl. 34 (1994) 256.
13. E. Tomboulis, Nucl. Phys. Proc. Suppl. 34 (1994) 192; Nucl. Phys. Proc. Suppl. 30 (1993) 549; Phys. Lett. B303 (1993) 103.
14. G. Bali, C. Schlichter, and K. Schilling, Phys. Rev. D51 (1995) 5165.
15. B. Bakker, M. Chernodub, and M. Polikarpov, hep-lat/9706007.
16. J. Greensite and M. Halpern, Phys. Rev. D27 (1983) 2545.
17. J. Greensite, Nucl. Phys. B315 (1989) 663.
18. T. Kovács and E. Tomboulis, proceedings of LATTICE 97, to appear in Nucl. Phys. Proc. Suppl.